CONFIDENTIAL*

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STPM 2006

MATHEMATICS T

PAPER 1

MATHEMATICS S

PAPER 1

Three Hours

PEPERIKSAAN PERCUBAAN STPM

DAERAH MUAR

2006

Instructions to candidates

*Answer **all** questions.*
All necessary working should be shown clearly.
Non-exact numerical answers may be given correct to three significant figures, or one decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

This question paper consists of 3 printed pages

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1. Using the laws of the algebra of sets, show that

$$A' \cap (B - A)' = (A \cup B)'$$ [4 marks]

2. If $y = \sin[\ln(1+x)]$, show that $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 0$. [4 marks]

3. The equation of a curve is

$$2x^2 + 8xy + 5y^2 + 3 = 0.$$

Find the equations of the two tangents which are parallel to the x-axis. [5 marks]

4. Find the sum to n terms of the series

$$3 - x + \frac{x^2}{3} - \frac{x^3}{3^2} + \ldots + \frac{(-1)^{r-1}x^{r-1}}{3^{r-2}} + \ldots$$

Given that the series is convergent, state the set of possible values of x, and the sum to infinity. [6 marks]

5. Solve the inequality $|5x - 7| > 2x + 1$. [5 marks]

6. The coordinates of the vertices O, A and B of a triangle are (0, 0) , (12, 0) and (0, 16) respectively.
   a) Verify that the point with coordinates (4, 4) is equidistant from the three sides of the triangle OAB. [5 marks]

   b) Obtain the equation of the inscribed circle of the triangles OAB. [4 marks]

7. a) Matrices P and Q are given as $P = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$, $Q = \begin{pmatrix} x & y \\ 1 & -1 \end{pmatrix}$, where x and y are real numbers. Find a diagonal matrix S such that $PSP^{-1} = Q$. [5 marks]

   b) If $M = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 0 & 2 \\ 1 & 1 & -1 \end{pmatrix}$, show that $M^3 = M$. Hence, find M^{20}. [5 marks]

8. Sketch the curves $y = x^2(5x - 6)$ and $y = x(4x - 5)$ on the same diagram. [3 marks]

Calculate the area of the region bounded by the curves. [5 marks]

9. The complex numbers z_1 and z_2 satisfy the equation $z^2 = -3 + 4i$

(a) Express z_1 and z_2 in the form $a + bi$, where a and b are real numbers. [6 marks]

(b) Represent z_1 and z_2 in an Argand diagram, [1 mark]

(c) For each of z_1 and z_2, find the modulus, and the argument in radians. [4 marks]

10. (a) The function f and g are given by

$$f(x) = e^{2x}, \qquad g(x) = 1 - x, \qquad x \in \Re.$$

(i) State the domains of f and g. [1 mark]

(ii) Find $f^{-1}(x)$ and hence obtain $(g \circ f^{-1})(x)$. [5 marks]

(b) If function h is defined as

$$h(x) = \begin{cases} Ax + 2, & -3 \le x \le 1 \\ x^2 + Bx + A, & 1 \le x \le 2 \\ 1 - \dfrac{1}{x}, & 2 \le x \le 5 \end{cases}$$

Find the values of A and B such that h is continuous in the interval $[3,5]$.

[5 marks]

11. Express $f(x) = \dfrac{3x^2 - 16x + 11}{(2-x)^2(1+4x)}$ in partial fractions. [5 marks]

Hence, find the expansion of $f(x)$ in ascending powers of x up to the term in x^3. [6 marks]

State the set of values of x which the expansion is valid. [2 marks]

12. It is given that $y = e^x$ and $y = 7 - 3x$.

(a) Sketch the graph of the two functions on the same coordinates axes. [3 marks]

(b) Use your graph in (a) to deduce a first approximation to the root of the equation $e^x = 7 - 3x$. [1 mark]

(c) Show that the equation $e^x = 7 - 3x$ has a root that lies between 1 and 1.5. [4 marks]

(d) By using the Newton-Raphson method, determine the root of the equation correct to three decimal places. [6 marks]